Seat No.	

S–1227

Total No. of Pages : 3

F.E. (Part - I) (Semester - I) (Revised) Examination, May - 2015

ENGINEERING MATHEMATICS - I

Sub. Code : 59177

Day and Date : Wednesday, 27 - 05 - 2015 **Total Marks : 100**

Time : 10.00 a.m. to 01.00 p.m.

Instructions :
1) **All the questions are compulsory.**
2) **Figures to the right indicate full marks.**
3) **Use of non-programmable calculator is allowed.**

SECTION - I

Q1) Attempt Any Three of the following : **[15]**

a) Reduce following matrix to normal form and hence determine the rank

$$A=\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

b) Test for consistency and if possible solve the equations $x + y + z = 3$; $x + 2y + 3z = 4$; $x + 4y + 9z = 6$.

c) Find λ and μ if the following equations has an infinite no. of solutions. $2x - 5y + 2z = 8$; $2x + 4y + 6z = 5$; $x + 2y + \lambda z = \mu$.

d) Use matrix method to solve the equations $x + 3y - 2z = 0$; $2x - y + 4z = 0$; $x - 11y + 14z = 0$.

Q2) Attempt Any Three of the following : **[15]**

a) Examine for linear dependence or independence of vectors [3 1 1]; [2 0 –1]; [4 2 1]. If dependant find the relation between them.

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b) Find the Eigen values of the matix $A=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and hence find Eigen values of A^T; (Adj-A) and A^{-1}.

c) Verify Caley–Hamilton's Theorem for the matrix $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

d) Find Eigen vector for least Eigen value of the matrix $A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.

Q3) Attempt Any Four of the following : **[20]**

a) Prove that $\sin^7\theta = \frac{1}{2^6}[35\sin\theta - 21\sin(3\theta) + 7\sin(5\theta) - \sin(7\theta)]$

b) Find all the roots of equation $x^{10} + 11x^5 + 10 = 0$

c) Prove that $\sinh^{-1} z = \log\left(z + \sqrt{z^2+1}\right)$.

d) If $\sin(x + iy) = [\cos\alpha + i\sin\alpha]$ then prove that $\sin^2\alpha = \cos^4 x$.

e) Separate into real and imaginary parts of $\cos^{-1}\left(\frac{3i}{4}\right)$.

SECTION - II

Q4) Attempt Any Three from the following : **[15]**

a) Expand $\log(1 + \sin x)$ by Maclaurians theorem.

b) Show that $e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{11x^4}{24} + \ldots\ldots\ldots\ldots$

c) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$.

d) Evaluate $\lim_{x\to 1}\left[\frac{x}{x-1} - \frac{1}{\log x}\right]$.

Q5) Attempt Any Four from the following : **[20]**

a) If $u = \log\left(\frac{x^2 + y^2}{xy}\right)$; Prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

b) If $z = f(x, y)$, where $x = e^u \cos v$, $y = e^u \sin v$, then prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$$

c) If $x = \frac{vw}{u}, y = \frac{wu}{v}, z = \frac{uv}{w}$ then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

d) If $f(x, y) = \left(50 - x^2 - y^2\right)^{1/2}$ find the approximate value of $f(3, 4) - f(2.9, 4.1)$

e) Find the extreme value of $xy(a - x - y)$, where $a > 0$.

Q6) Attempt Any Three from the following : **[15]**

a) Apply Gauss elimination method to solve $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$.

b) Solve by Jacobi's iteration method $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ upto fifth iteration.

c) Solve by Gauss-Seidal iterative method $10x + y + z = 12$, $2x + 10y + z = 13$; $x + y + 5z = 7$.

d) Find the largest Eigen value of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ using powers method by taking $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, upto fifth iteration.

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