

Seat No.	
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First Year Engineering (All Branches) (Semester - I & II)

Examination, May - 2017

ENGINEERING MATHEMATICS - I

Sub. Code : 59177

Day and Date : Saturday, 20 - 05 - 2017

Total Marks : 100

Time : 10.00 a.m. to 1.00 p.m.

- Instructions :**
- 1) All questions are compulsory.
 - 2) Figures to the right indicate full marks.
 - 3) Use of non-programmable calculator is allowed.

SECTION - I

Q1) Attempt any "Three" of the following:

[15]

- a) Find the rank of matrix A by reducing to its normal form if

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

- b) Test for consistency and if possible solve the equations $x + y + z = 3$; $x + 2y + 3z = 4$; $x + 4y + 9z = 6$.
- c) Investigate values of λ and μ if the equations has an infinite no. of solutions.
 $2x - 5y + 2z = 8$; $2x + 4y + 6z = 5$; $x + 2y + \lambda x = \mu$.
- d) Use matrix method to solve the equations
 $2x - y + 3z = 0$; $3x + 2y + z = 0$; $x - 4y + 5z = 0$.

P.T.O.

Q2) Attempt any "Three" of the following:

- a) Examine for linear dependence or independence of vectors $[1 \ 1 \ 1 \ 3]$; $[1 \ 2 \ 3 \ 4]$; $[2 \ 3 \ 4 \ 7]$. If dependent find the relation between them.
- b) Find eigen vector corresponding to the greatest eigen value of the matrix

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

- c) Verify Caley-Hamilton's theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

- d) Find eigen values of the matrices:

- i) A^{-1}
 ii) $(\text{Adj. } A)$
 iii) $4A$

if the matrix $A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$.

[20]

Q3) Attempt any "Four" of the following:

- a) Find all the roots of $(1 + i)^{1/5}$.
- b) Prove that $\left[\frac{\sin(6\theta)}{\sin \theta} \right] = 32 \cos^5 \theta - 32 \cos^3 \theta + 6 \cos \theta$.
- c) Separate into real and imaginary parts of $\tan^{-1}(\alpha + i\beta)$.
- d) If $\sin(\theta + i\phi) = r[\cos \alpha + i \sin \alpha]$ then prove that

$$r^2 = \frac{1}{2} [\cosh(2\phi) - \cos(2\theta)]$$

- e) Solve the equation completely

$$x^8 + x^5 + x^3 + 1 = 0.$$

SECTION - II

Q4) Attempt Any Three of the following:

[15]

- a) Expand $\log(1 + e^x)$ by Maclaurian's theorem up to the term x^4 .
- b) Expand $(1+x)^{1/x}$ up to the term containing x^2 .
- c) Expand $\log x$ in powers of $(x-2)$.
- d) Evaluate $\lim_{x \rightarrow 1} \frac{xe^x - \log(1+x)}{x^2}$.

Q5) Attempt Any Four of the following:

[20]

- a) If $u = e^{xyz}$; find $\frac{\partial^3 u}{\partial x \partial y \partial z}$.
- b) If $z = f(x, y)$ and $x = uv, y = u^2 - v^2$ then prove that

$$\frac{\partial z}{\partial y} = \frac{1}{2(u^2 + v^2)} \left[u \frac{\partial z}{\partial u} - v \frac{\partial z}{\partial v} \right]$$

- c) If $\sin u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{400} \tan u [\tan^2 u - 19].$$

- d) Find $[(3.82)^2 + 2(2.1)^3]^{1/5}$ approximately by using theory of approximation.
- e) Find the maximum and minimum value of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

Q6) Attempt Any Three of the following:

a) Solve by Gauss elimination method

$$3x - y + 2z = 12; x + 2y + 3z = 11; 2x - 2y - z = 2.$$

b) Solve by Jacobi method (carry out five iterations)

$$6x + 3y + 12z = 35; 8x - 3y + 2z = 20; 4x + 11y - z = 33.$$

c) Solve by Gauss-Seidal method.

$$27x + 6y - z = 85; 6x + 15y + z = 72; x + y + 54z = 110$$

d) Find largest eigen value of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by power method with

$$X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ as the initial eigen vector.}$$